Retrieving implied financial networks from bank balance sheet and market data

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Conference on Network Models and Stress Testing for Financial Stability

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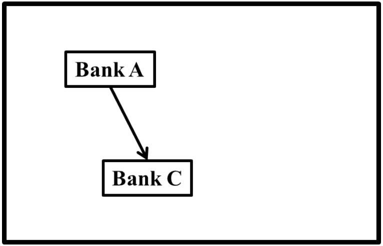
Outline

Motivation

- Interconnectedness is a structural vulnerability (Christensen, I. et al, 2015):
  - In normal times, makes the financial system more resillient by providing risk-sharing opportunities;
  - In times of stress, can allow for distress propagation.
- Lessons from the 2007–09 crisis:
  - Reserve primary fund "breaks the buck" due to its exposure to Lehman's CP;
  - AIG bailout.
- Simulations used to assess the risk of contagion frequently employ a network of exposures (e.g.,MFRAF):
  - MFRAF: Interbank and Major Exposures Return.

Motivation (2)

- Market participants only have a partial knowledge of these exposures.
  - Example: simple interbank loan



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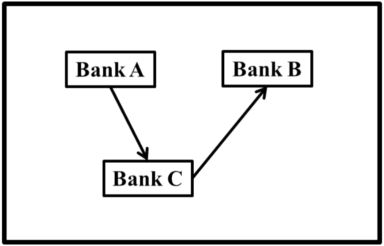

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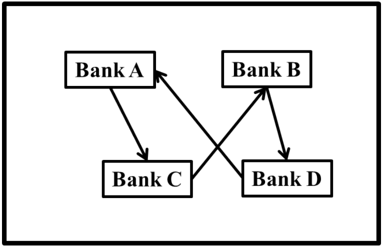


"Knowing your ultimate counterparty risk then becomes like solving a high-dimension Sudoku puzzle" Haldane (2009).

Motivation (3)

- A motivating example:

	A	B	C	D	Total
A	0	?	?	?	50
B	?	0	?	?	5
C	?	?	0	?	20
D	?	?	?	0	20
Total	20	50	20	5	

? unknown;
– known.

Motivation (4)

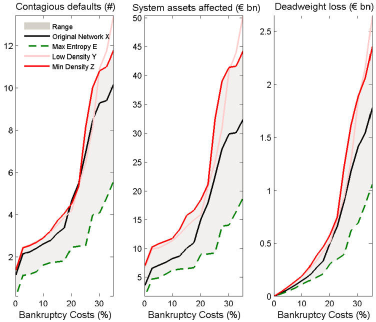
- Potential solutions:

	A	B	C	D	Total
A	0	31	15	4	50
B	3	0	2	0	5
C	9	10	0	1	20
D	8	9	3	0	20
Total	20	50	20	5	

	A	B	C	D	Total
A	0	30	20	0	50
B	0	0	0	5	5
C	0	20	0	0	20
D	20	0	0	0	20
Total	20	50	20	5	

Motivation (5)

- Standard method: Maximum Entropy (ME) approach - divide exposures equally among counterparties.
- Limitations:
  - Not informed by market participants' beliefs w.r.t. the distribution of these exposures;
  - Real data shows that ME produces biased results.



Source: Fig 6., Anand et al. (2014)

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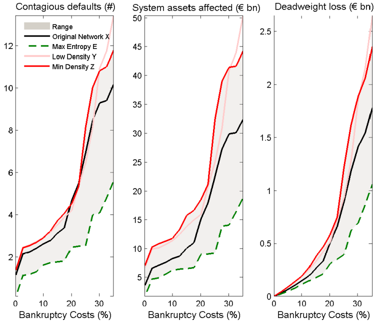

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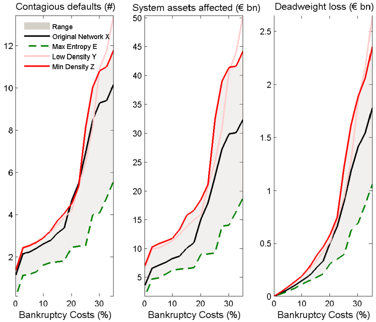

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Proposed Contribution

- Use market signals of network-dependent contingent claims to infer the implied network of exposures:
  - Pricing model that takes into account the network of exposures: implied price;
  - Data on market signals of contingent claims: observed prices;
  - Find the network(s) that is(are) consistent with the observed prices and aggregate exposures.

Literature Review

- Reconstructing networks from partial information: Anand et al. (2017)
- Uncertainty in financial networks: Caballero and Simsek (2013) and Li et al. (2016);
- Asset pricing in financial networks: Eisenberg and Noe (2001), Egloff et al. (2007), Gouriéroux et al. (2013) and Barucca et al. (2016), and Abbassi et al. (2017).

Outline

1. Motivation
2. Implied Networks
3. Simulations
4. Empirical Application

The Pricing Model

- Consider a stylized balance sheet:

Assets	*Liabilities*
EA_i	EL_i
	IL_i
IA_i	EQ_i

EA - external assets; *IA* - internal assets; *EL* - external liabilities; *IL* - internal liabilities; *EQ* - equity.

The Pricing Model (2)

- Similar to the standard Merton (1974) model:
  - $\frac{dEA_i(t)}{EA_i} = \mu_i dt + \sigma_i dZ_i(t)$, where Z_i is a one-dimensional Brownian motion.
- Extension:
  - $IA_j(T) = \sum_{i=1}^{n} p_i^\star m_{ij}$, where m_{ij} is the exposure of bank j to bank i and $p_i^\star$ is the fraction of the exposure borrower i is able to repay at maturity.

Assumption

EL_i have the same priority in liquidation as IL_i.

The Pricing Model (3)

- Find the clearing payment vector:

$$p^{\star} = min\left\{\mathbf{1}, max\left[\left(\mathbf{M}'p^{\star} + \mathbf{e}\right) \odot \left(\frac{1}{d_1}, \ldots, \frac{1}{d_n}\right), 0\right]\right\},$$

where
$e = EA(T)$; $\mathbf{M}$ is the liabilities matrix (or network);
$d_i = \sum_j m_{ij} + EL_i$ and $\odot$ is the Hadamard product.

Assumption

Limited liability

Assumption

Proportional repayment

The Pricing Model (4)

- Example (revisited)

	A	B	C	D	Total
A	0	30	20	0	50
B	0	0	0	5	5
C	0	20	0	0	20
D	20	0	0	0	20
Total	20	50	20	5	

- Eisenberg and Noe (2001) propose an algorithm to find $p^{\star}$.
- A owes \$30 to B and \$20 to C, but D owes \$20 to A.
- If $e_A = \$20$,then, assuming D is solvent, A only has \$40 to pay B and C.
- Thus, A can only pay 80 cents on the dollar owed (fundamental default).
- If $e_C = 0$, then C will also only be able to pay 80 cents on the dollar (default by contagion).

The Pricing Model (5)

- I allow for bankruptcy costs as in Rogers and Veraart (2013):
  - Failed banks' external assets lose α of their value.
- Price contingent claims by Monte Carlo simulation since a closed form solution does not exist:
  1. Sample a random path for EA_i in a risk-neutral world;
  2. Find $p^\star$ for each sample path;
  3. Repeat steps 1. and 2. K times;
  4. Given $p^\star$ find the payoff of the contingent claim for each sample path and then compute the average payoff;
  5. Discount the expected payoff using the risk-free rate to obtain the theoretical price.

Implied Networks

- Example of a zero-coupon bond with \$1 face value:
  - Set of implied networks:

$$\mathbb{IN} := \underset{\tilde{\mathbf{M}} \in \mathbb{F}}{argmin} \left\{ \Xi \left(\tilde{\mathbf{M}} \mid \theta \right) \right\},$$

where

$$\Xi \left(\tilde{\mathbf{M}} \mid \theta \right) := \frac{1}{n} \left[p_t^{obs} - e^{-r_f \tau} E_t^Q \left(\mathbf{p}^{\star} \left(\tilde{\mathbf{M}} \mid \theta \right) \right) \right]' W \left[p_t^{obs} - e^{-r_f \tau} E_t^Q \left(\mathbf{p}^{\star} \left(\tilde{\mathbf{M}} \mid \theta \right) \right) \right],$$

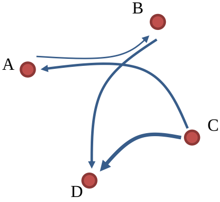
with

$$W = \begin{bmatrix} \frac{TA_1}{\sum_{j=1}^{n} TA_j} & 0 & \dots & 0 \\ 0 & \frac{TA_2}{\sum_{j=1}^{n} TA_j} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \frac{TA_n}{\sum_{j=1}^{n} TA_j} \end{bmatrix}, \ \theta = (TA, \Sigma).$$

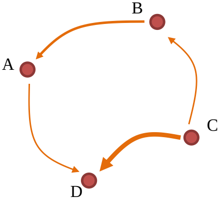
Heuristic approach to network optimization

1. Use the ME network as a starting point;
2. Generate *ngen* "mutations" of the ME matrix. This set of matrices is referred to as the children set;
3. Evaluate the fitness, i.e., of all matrices generated in 2.;
4. Preserve the *npar* matrices with the lowest weigthed mean squared error. This set of matrices is referred to as the set of parents;
5. Create *ngen* "mutations" based on the matrices identified in 4. and proceed as in 3.;
6. Repeat steps 4 and 5 until the fitness measure shows a high enough improvement over the fitness measure obtained under ME.

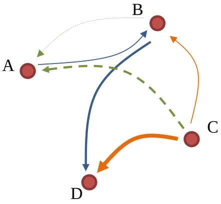
Heuristic approach to network optimization (2)



Parent 1



Parent 2



Child 1

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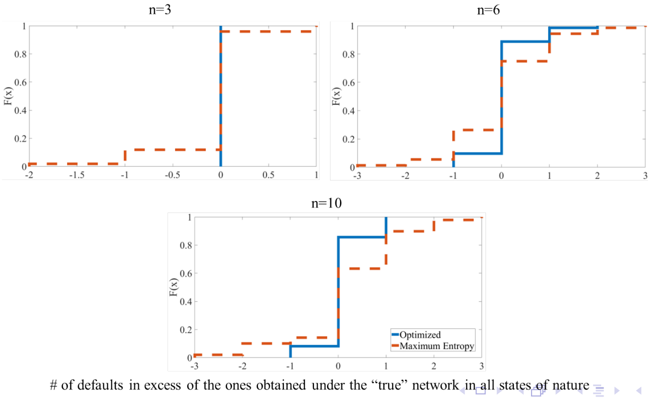
Simulation Results

- Simulate the “true” network implied by market prices:
  - Based on the total exposures: obtain the ME network;
  - Based on the implied price: obtain the optimized/implied network.

Simulation Results (2)

Setup: bankruptcy costs = 5% (i.e., $\alpha = 0.95$).

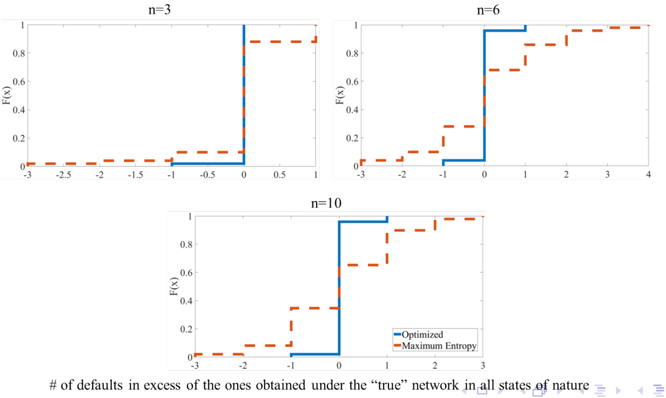
Figure: CDF of the deviations in the number of defaults vis-a-vis the "true" network



Simulation Results (3)

Setup: bankruptcy costs = 15% (i.e., $\alpha = 0.85$).

Figure: CDF of the deviations in the number of defaults vis-a-vis the "true" network



Simulation Results (4)

Setup: noisy prices for $n = 6$ and $\alpha = 0.95$.

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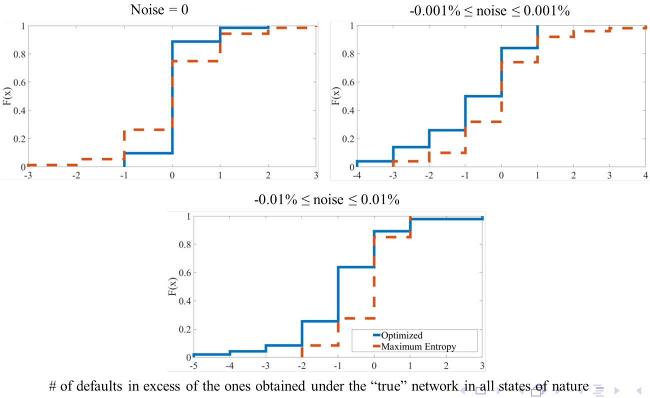


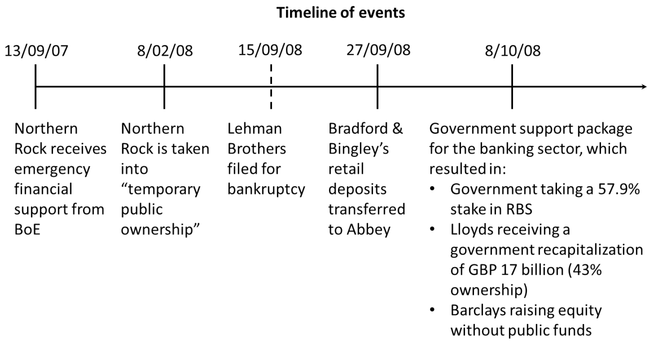
Outline

Data

- 4 most systemic UK banks (part of the G-SIB list) - HSBC, RBS, BARC and STAN.
- Data on aggregate exposures referring to the 2007–09 crisis period:
  - Internal assets - “Loans and advances to banks”;
  - Internal liabilities - “Deposits by banks”.
- Market signals: 5-year Credit Default Swap (CDS) spreads;
- Calibration: following Chatterjee (2013) bankruptcy costs set at 10%.

Data (2)

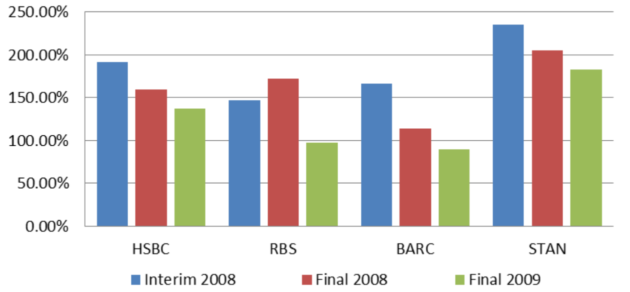
Timeline of the financial crisis in the UK



Source: author's summary based on House of Commons (2009).

Data (3)

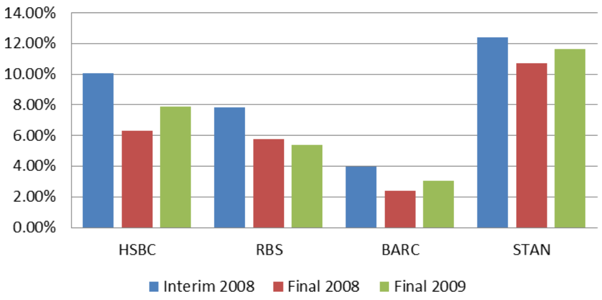
Ratio of internal (interbank) assets to equity (book values)



Source: Banks' annual reports.

- Interbank assets/book-value equity between 90 and 230%.

Data (4)
Ratio of internal (interbank) assets to total assets (book values)

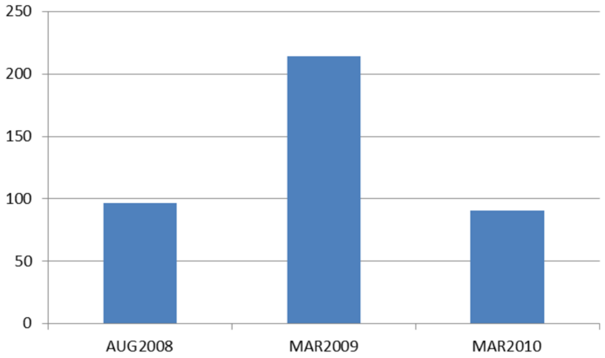


Source: Banks' annual reports.

- Interbank assets/book value assets between 3 and 12%.

Data (5)

5-Year CDS spreads (in bps)

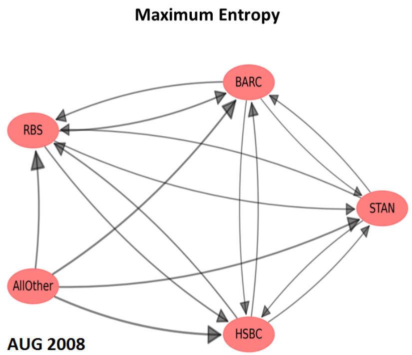


Source: Bloomberg.

Estimation

- External assets stochastic processes' parameters estimated by Maximum Likelihood as in Duan (1994), Duan (2000).
- Consistency among total, external and internal assets obtained via a numerical fixed-point approach.

Results



Note: Edges' width represent the log of exposures. A -> B represents exposure of A to B. Appendix

Results (2)

Relative differences expressed in percentage terms of the ME exposures

AUG2008	HSBC	RBS	BARC	STAN	Liabilities
HSBC		-18	28	221	
RBS	-8		-18	-83	
BARC	8.6	17		-87	
STAN	15	-3	-6		

DEC2008	HSBC	RBS	BARC	STAN
HSBC		19	2	48
RBS	-19		8	-42
BARC	27	-33		-9
STAN	-33	71	-80	

DEC2009	HSBC	RBS	BARC	STAN	Liabilities
HSBC		7	6	-68	
RBS	-22		-1	66	
BARC	29	4		-9	
STAN	-15	-58	-27		

Results (3)

- The network structure seems to be relevant for the pricing of risk:
  - (Roughly) 40% reduction in the weigthed mean squared error under implied/optimized network in comparison to ME in some of the time slices;
  - Consistent with market participants having more information than simply aggregate exposures.

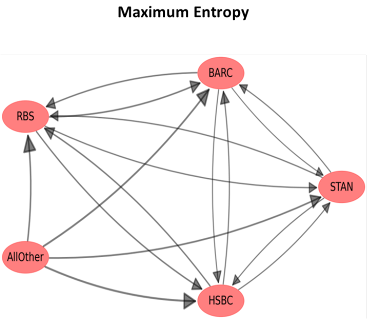
Results (4)

- Improvement over ME varies over time:
  - 40% improvement when looking at the AUG2008 and DEC2009 time slices, but declines to 25% improvement for the DEC2008 time slice;
  - Potential explanation: when the overall riskiness of banks increases so does the inability to distinguish bilateral exposures.
    - Argument consistent with CDS spreads (highest for the DEC2008 slice).

Summary

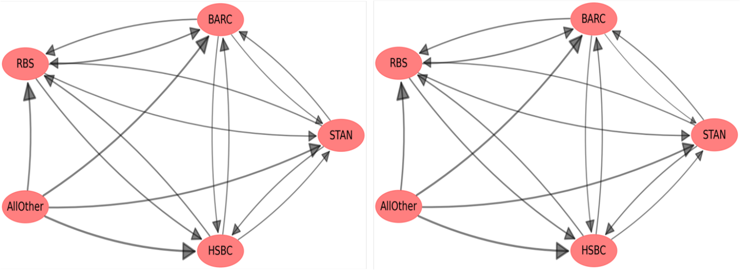
- Proposed a method to retrieve the set of implied networks from market data:
  - Value-added: Inform re: beliefs and behavioral reactions of market participants towards an institution.
- Increases in overall riskiness may play a role in market participants' ability to discern bilateral exposures.
- Limitations:
  - Partial identification;
  - Only as good as the underlying pricing model;
  - Only as good as the market data reflect counterparty risk.

Appendix
August 2008



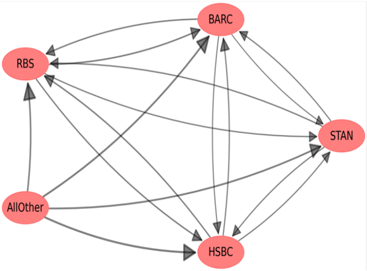
Appendix
December 2008

Maximum Entropy | **Implied Network**



Appendix
December 2009

Maximum Entropy



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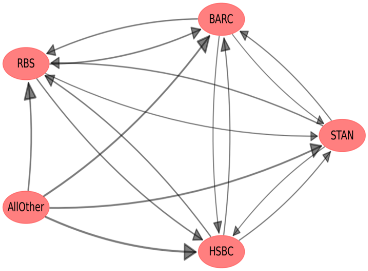

Return